Project M@th Desktop

Reinhard SIMONOVITS, HAK Graz

M@th Desktop (Abkürzung MD) ist eine interaktive, multimediale und internetunterstützte Mathematiklernsoftware für AHS Oberstufe und BHS. MD basiert auf dem Programmpaket Mathematica.
Das didaktische Konzept ist das Toolkonzept wie man es z.B. von Excel gewohnt ist.

Das Arbeiten mit MD erfolgt hauptsächlich über Paletten. Die Arbeitsblätter sind so gestaltet, dass der Benützer Text, Rechnungen, Graphiken und Internetlinks
leicht einbauen kann. Ein Author Tool für Lehrer ist vorhanden.

Im konkreten werden Bsp aus der Differentialrechnung gezeigt.

Univ. Prof. Dr. Bernd Thaller, bernd.thaller@kfunigraz.ac.at, vom Institut für Mathematik, Uni Graz und Dr. Reinhard Simonovits, Reinhard.Simonovits@kfunigraz.ac.at, von der Handelsakademie Grazbachgasse, Graz, bilden die Projektleitung von MD.
Dort erhalten sie auch weitere Informationen.

Anbei Ausschnitte von der Lehrerversion des Arbeitsblattes ueber die mittlere Geschwindigkeit.

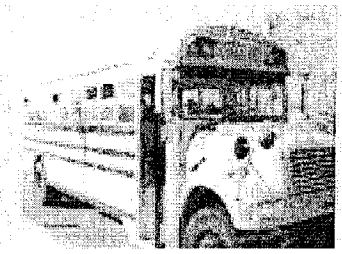
The Difference Quotient

Average Velocity

Get Palette !

1

THE SCHOOL BUS MOVIE : Visualizing Velocity



1.1 The School bus stops again

You leave your school bus. The bus goes away, but slows down and stops because Jennifer forgot to get out.

It takes 4 seconds for the bus to slow to a stop.
Analyze the average velocity of the bus between the point when it begins to slow down and when it stops.

1.2 The Average Velocity Movie

Let's experiment a little... Try different **Δt**´s between 0.2s and 2s.

Keep your eye on the plot label at the top of the graph. Note the change in value of the average velocity as you *increase* or *decrease* the value of Δt.

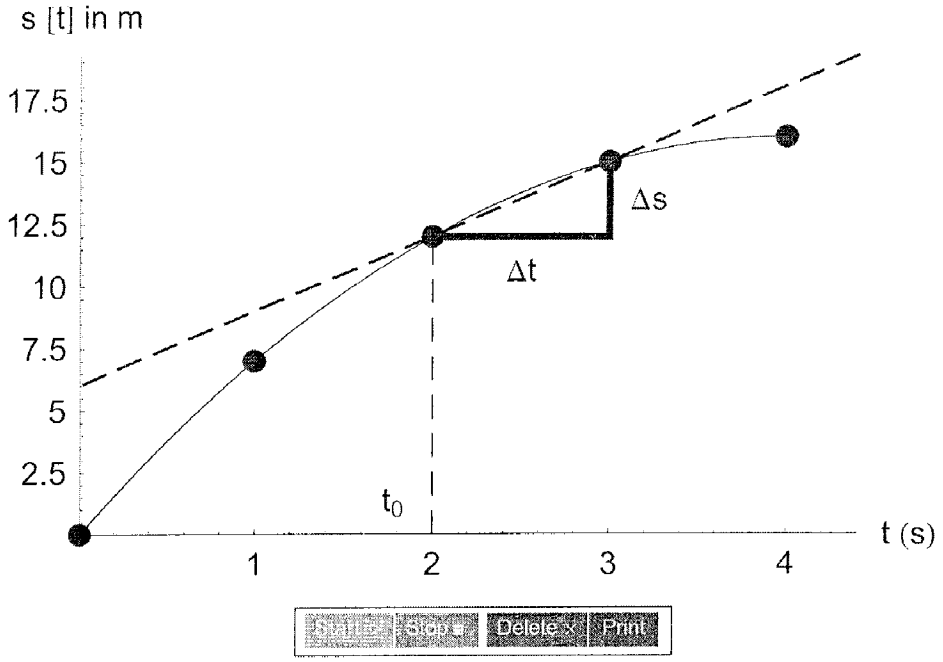
Click the START *button to produce the movie.*

Input ▷

```
Δt = 1.0;
DifferenceQuotientMovie[Δt]
```

$(s[t_0+\Delta t] - s[t_0]) / \Delta t = +0010.80$ km / h

$t_0 = 02.00$ s



Questions about the Movie

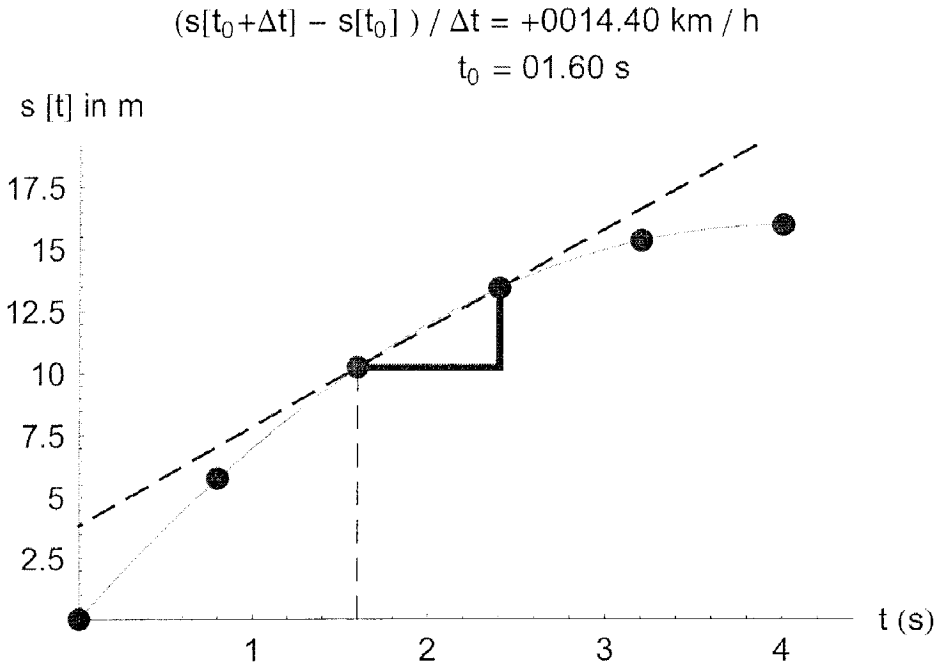
DEFINITION : Difference Quotient

$$\frac{s[t_0 + \Delta t] - s[t_0]}{\Delta t}$$

2.1 Graphical Explanation

$$\text{Average Velocity} = \frac{\text{Distance moved}}{\text{Time elapsed}} = \frac{s[t_0 + \Delta t] - s[t_0]}{\Delta t}$$

= Slope of the green dashed secant line



2.2 Definition

A difference quotient is a number measuring the ***average rate of change*** of a function over a given argument interval. Get it?

In our case the difference quotient is applied to physics.

Here it is the change in the distance function s[t] over the time interval Δt to a given time t_0 .This is called the ***average velocity***.

$$\text{Difference Quotient} = \frac{s[t_0 + \Delta t] - s[t_0]}{\Delta t}$$

EXAMPLE : Step by Step

3.1 Step by Step

Example: A Mercedes slows down at a traffic light. The distance as a function of time is given by $s[t] = 8\,t - t^2$.
What is the Mercedes' average velocity in km/h at $t_0 = 2.1$s over the interval $\Delta t = 0.2$s?

Solution: Use the formula for the difference quotient. Don't forget to multiply your result with 3.6 (km/h!)

Click the Difference Qu button.

```
Clear[s, t];
s[t_] = 8 t - t^2;
```

Input ▹

$$\frac{s[2.1 + 0.2] - s[2.1]}{0.2} * 3.6$$

```
12.96
```

Now press the Answer button to summarize your result.

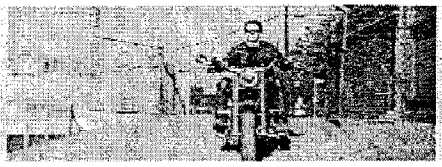
Answer: The velocity of the Mercedes is about 13 km/h.

More details ▼.

EXERCISE SECTION
Test Your Knowledge

Practice

Schwarzenegger Problems



Arnold Schwarzenegger escapes on a motorbyce. He puts the pedal to the metal. What is his average velocity after t_0 = 2.4s?

Δt = 0.8s, s[t] = $3.5t^2$.
Solution: 19.6 m/s.

*Click the Difference Qu button. Select the **Answer** button of the palette to summarize your results.*

```
Clear[s, t];
s[t_] = 3.5 t^2;
```

Input ▹

$$\frac{s[2.4 + 0.8] - s[2.4]}{0.8}$$

```
19.6
```

Answer: The velocity after 2.4s over 0.8s is 19.6 m/s.

Arnold is in a bad situation.150 m in front of him a truck crosses the road. He brakes. What is his average velocity at t_0 = 0.5s over Δt = 0.05s and t_0= 0.9s over Δt = 0.05s.

$s[t] = 7t - 3.5t^2$.
Solution: 3.3 m/s, 0.5 m/s.

Click the Difference Qu button. Select the ***Answer*** *button of the palette to summarize your results.*

```
Clear[s, t];
s[t_] = 7 t - 3.5 t^2;
```

Input ▷

$$\frac{s[0.5 + 0.05] - s[0.5]}{0.05}$$

3.325

```
Clear[s, t];
s[t_] = 7 t - 3.5 t^2;
```

Input ▷

$$\frac{s[0.9 + 0.05] - s[0.9]}{0.05}$$

0.525

Answer: The velocity after 0.5s over 0.05s is 3.325 m/s, after 0.9s 0.525 m/s.

* Difficult. Continuing Arnold´s bad situation (4.2).
Do you think he has good brakes - or is this a computer animation?

Hint: Apply the concept of a difference quotient on velocities. The result is the average acceleration a in m/s^2.
Name the average velocity at t_0 = 0.5s v1, at t_0= 0.9s v2. What is Δt?
Calculate a = (v1 - v2) / Δt.
Compare your result with good brakes on dry ground: a = 7 $\frac{m}{s^2}$.
Solution: 7 m/s^2 .

Select the ***Answer*** *button of the palette to summarize your results.*

```
v1 = 3.325;
v2 = 0.525;
Δt = 0.4;
```

Input ▷

$$\frac{v1 - v2}{\Delta t}$$

7.

Answer : Arnold's brakes are ok. Probably no computer animation was done.

Arnold reverses after slowing to a stop.
What is his average velocity after t_0 = 1.2s. Why is the average velocity negative?

$\Delta t = 0.05s$, $s[t] = 7t - 3.5t^2$..
Solution: -1.575 m/s.

Click the Difference Qu button. Select the ***Answer*** *button of the palette to summarize your results.*

```
Clear[s, t];
s[t_] = 7 t - 3.5 t^2;
```

Input ▷

$$\frac{s[1.2 + 0.05] - s[1.2]}{0.05}$$

```
-1.575
```

Answer:The average velocity after 1.2s over 0.05s is -1.575 m/s. Arnold goes back and this is pointed out by the minus sign.

Tennis Ball and Balcony

Computer Lab

Test Your Knowledge, Keywords

SUMMARY & INTERNET **Resources**

SUMMARY

The difference quotient $(s[t_0 + \Delta t] - s[t_0]) / \Delta t$ is a method, **how to estimate velocity e.g. of a car, tennis ball, etc. numerically.**

The formula gives you the average rate of change **of the distance** s **over the time interval** Δt.

The result of this calculation is the average velocity **given in m/s.**

In general, you get better results **in everyday life, if** Δt **is only a fraction of a second.**

INTERNET

A list of servers with a server description providing Internet resources on difference quotient is given at:

www.math.desktop.at/english/DifferenceQuotient

SOURCES